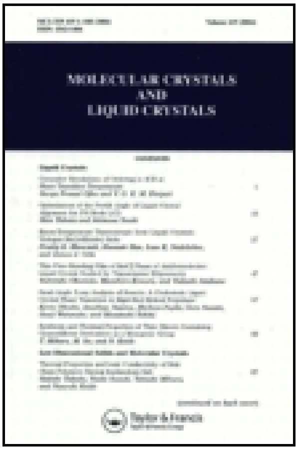

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Nematic Deformation in Homeotropically-Aligned Liquid Crystal Microlens and its Optical Properties

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The experimental and theoretical investigations of the director deformation of homeotropically aligned nematic in axially – symmetrical electrical field of an asymmetrical liquid crystal microlens (LCM) were carried out. The calculation results of microlenses phase profiles and focal distance are in satisfactory accordance with experimental values.

Keywords: liquid crystal microlens; focal distance

INTRODUCTION

The effect of nematic molecules realignment in an axially symmetrical electrical field created by special geometry of electrodes is used for control the light wave phase in liquid crystal (LC) microlenses. The experimental investigations results of some optical properties LC-microlenses with homeotropically aligned nematic are represented in [1]. The calculation of the director deformation in the axially symmetrical electrical field was carried out earlier only for small nematic deformation angles [2, 3]. The purpose of the present papers is the research of the homeotropically aligned nematic deformation in axially symmetrical electrical fields at large angles of a deformation.

EXPERIMENTAL RESULTS

Asymmetrical LC-microlens was described first in paper [4]. It represents a cell, upper glass substrate of which is coated by layer of indium oxide, and lower one is coated by thin opaque layer of chrome. In lower layer were created holes by means of photolithography. In our

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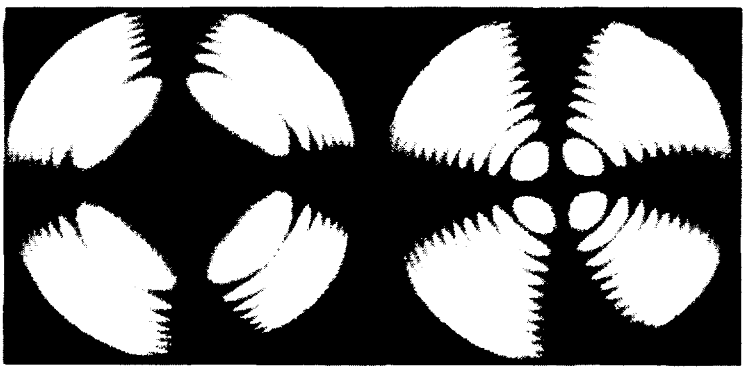
research the diameters of holes (L) were equal to 180; 380; 600 and 780 microns. A thickness of a LC-layer (d) is 50 microns. The nematic represents a mixture of MBBA and EBBA ($\Delta\varepsilon = -0{,}43; \Delta n = 0{,}23; n_o = 1{,}52$). Homeotropical alignment was reached by the volumetric addition of lecithin (0,1 %). The alternate voltage of frequency 2 kHz was given. The observations of an interference pattern were carried out with the help of polarizing microscope ($\lambda = 0{,}637\ \mu m$); the focal distance was measured by means of microscope [1]. At small voltages in a LC-cell the realignment of molecules is beginning from the edges of the hole. The interferential rings being the consiquence of the interference of ordinary and extraordinary waves (Figure 1 a) are visible in this area in the polarizing microscope. The deformation region increases and step by step reaches the center of the hole with increased voltages (Figure 1 b).



a) b)

FIGURE 1 Interference patterns observed in LC-microlens of diameter 380 microns at different voltages: a) 7V, b) 10V

The interferograms used for a construction of a phase delay profiles at different voltages. They are shown on a Figure 2 for a microlens of diameter 380 microns at 5, 7, 10, 14, 18 voltage values. The experiments have shown that the process of nematic realignment in LC-microlens passes through five stages with different shapes of a phase profile. At small voltages near to a center of the hole the profile of a phase delay has a plateau, which one decreases with increased of voltages. The size of the plateau decreases at increased U. The profile of the phase delay becomes parabolic and the microlens becomes spherical (stage two) at certain voltage U_p (which is different for various microlenses). The voltage U_p is increased with growth of a size of a lens and in our experiments varies from 7 to 18 volts. The deformation in LCM passes through following three stages: conic, with disclination in a center, spiralized (with a quasilinear dependence of a phase from a profile) at further increase of voltage.

The measurements of focal distances of microlenses are carried out. It is revealed that the focal distance is negative, i.e. the microlenses with homeotropical alignment possess of defocusing properties. From the begining the focal distance decreases

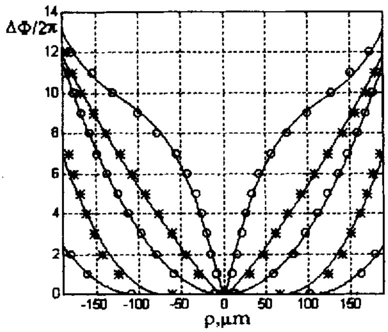


FIGURE 2 Profiles of the phase delay with homeotropical align of diametrs 380 microns at 5, 7, 10, 12, 15 voltage values.

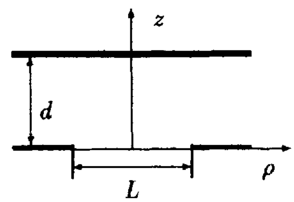


FIGURE 3 The scheme of a LC-cell.

reaching the minimum value at voltage U_p, and then increases at increased voltages [1]. LC-microlens possesses the best optical properties at voltage U_p. The deviation of phase profile of the LC-microlenses with $L/d \leq 7$ from a parabola does not exceed $\lambda/4$, i.e. the optical properties LC-microlens are in conformity with the criterion of Rayligh.

THE THEORY AND CALCULATION

The calculation of the director distribution in an inhomogeneous electrical field in the assumption of small angles of deformation was considered in papers [2, 3]. The analitical solution of the Laplace equation for an electrostatic potential and Euler-Lagrange equation for a deformation angle θ were obtained also just there. The method of obtaining of analytical solutions of two-dimensional nematic deformation in Cartesian coordinate system (x, y) was developed in [5]. However, similar task in axially symmetrical electrical field at large deformation angles doesn't have an analitical solution now. Therefore it was resolved numerically. Let's consider equations system describing an electrical field $\vec{\mathbf{E}}\ (\rho, z)$ and the nematic director distribution $\vec{\mathbf{n}}\ (\rho,\ z)$. The configuration of electrodes and coordinate system are illustrated in Figure 3. The symmetry of the task allows to consider a two-dimensional distribution of vectors $\vec{\mathbf{n}}$ and $\vec{\mathbf{E}}$. The vector $\vec{\mathbf{E}}$ is determined by the components in a cylindrical coordinate system $(\rho,\ \varphi,\ z)$:

$$\vec{\mathbf{E}} = (E_\rho,\ 0,\ E_z) = E(\sin\psi,\ 0,\ \cos\psi).$$

The dielectric anisotropy is small for a LC material used in our experiments: $\delta = \frac{\varepsilon_{\|} - \varepsilon_{\perp}}{\varepsilon_{\|} + \varepsilon_{\perp}} = 0,09$, where $\varepsilon_{\|}$, $\varepsilon_{\perp}$ – the components of dielectric tensor. In this case it is possible to write electrostatic equations as the two-dimensional Laplace equation for an electrostatic potential

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial}{\partial\rho}u) + \frac{\partial^2}{\partial z^2}u = 0. \tag{1}$$

The boundary conditions for this equation are the next:

$$\begin{aligned} u(\rho,0) &= \begin{cases} \varphi(\rho), & \rho < L/2 \\ 0, & \rho \geq L/2 \end{cases}, & \frac{\partial u(0,z)}{\partial\rho} &= 0, \\ u(\rho,d) &= U, & u(z,\infty) &= \frac{U}{d}z. \end{aligned} \tag{2}$$

Function $\varphi(\rho)$ giving values of a potential in an hole of the lower electrode of a lens is determined from a solution of more general boundary (exterior relatively LC area) task. The boundary conditions for which was defined proceed from the most common suppositions: $u_\infty = 0$. A polarized charge appears on the glass surface as the result of difference of dielectric penetrability both glass (ε_{gl}) and nematic (ε_{lc}). It was taken into account at calculation of function $\varphi(\rho)$. The coupling conditions for electrical potential on the border of two mediums are as follows:

$$\varepsilon_{lc}\frac{\partial u_{lc}}{\partial z} = \varepsilon_{gl}\frac{\partial u_{gl}}{\partial z}.$$

Let's consider now the equation describing the nematic behaviour in an electrical field. In homeotropical LC-microlens the electrical field is axially symmetrical. Therefore there is no twist-deformation. Distribution of nematic director is described by the equation:

$$\vec{\mathbf{n}} \times [(K_{11} - K_{33}) \vec{\nabla} (\vec{\nabla}\vec{\mathbf{n}}) + K_{33}\nabla^2 \vec{\mathbf{n}} + \Delta\varepsilon\varepsilon_0 \vec{\mathbf{E}} (\vec{\mathbf{n}} \cdot \vec{\mathbf{E}})] = 0, \quad (3)$$

where K_{11}, K_{33} – Francs elastic constant, $\Delta\varepsilon$ – dielectric anisotropy of nematic, ε_0 – dielectric constant.

Let's assume an one-constant approximation $K_{11} = K_{33} = K$. Then equation of the Euler - Lagrange (3) is performed to more simple form. The director $\vec{\mathbf{n}}$ has components in a cylindrical coordinate system: $\vec{\mathbf{n}} = (n_\rho,\ n_\varphi,\ n_z) = (\sin\theta,\ 0,\ \cos\theta)$, $\theta = \theta(\rho,\ z)$, where θ – the angle between a vector of the director and axes z. Then equation (3) is presented as the follows:

$$K\nabla^2\theta - \Delta\varepsilon\varepsilon_0 E^2 \sin 2(\theta - \psi) = 0, \quad (4)$$

Numerical values of elastic constants are known less precisely than their ratio and threshold voltage, $U_0 = \pi \left[\frac{K_{33}}{\Delta\varepsilon\varepsilon_0}\right]^{1/2}$, which may be measured with necessary precision in homogenious electrical field by means of experiment. Therefore for a numerical solution the equation (4) is reduced to next

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial}{\partial\rho}\theta) + \frac{\partial^2}{\partial z^2}\theta + \left[\frac{\pi E(\rho, z)}{U_0}\right]^2 \sin 2(\theta - \psi) = 0. \quad (5)$$

This equation is solved in rectangular area $(0 \le z \le d,\ 0 \le \rho < \infty)$ with boundary conditions taking into consideration homeotropical alignment of liquid crystal molecules on glass substrates ($\theta = 0°$) and rotational symmetry of a lens:

$$\theta(\rho)|_{z=0} = 0, \quad \theta(\rho)|_{z=d} = 0, \quad \frac{\partial\theta}{\partial z}|_{\rho=0} = 0, \quad \frac{\partial\theta}{\partial z}|_{\rho=\infty} = 0. \quad (6)$$

The set of equations (1) and (5) with boundary conditions (2) and (6) form a mixed boundary task. The last one was solved numerically by means of a difference method.

THE ANALYSIS OF AN OBTAINED SOLUTION

All parameters values were taken conforming experimental at a solving given task. Equipotentials of electrical field in LC-microlens of diameter 600 μm at voltage $U = 14\ V$ are shown on a Figure 4. Equipotentials have been traced both in LC area and in glass area. A jog of equipotential lines caused by different dielectrical constants of liquid crystal and glass is visible on the border between them. Then one can see the field near the edge of the lower electrode has maximal inhomogeneity.

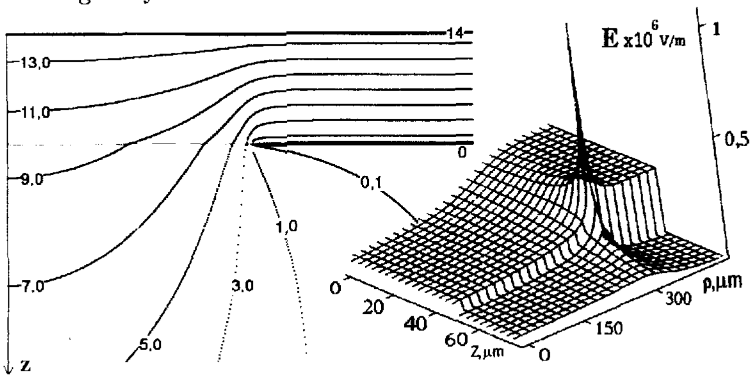


FIGURE 4 The equipotential lines and electrical field are traced into and out of the limits of LC under working voltage 14 V.

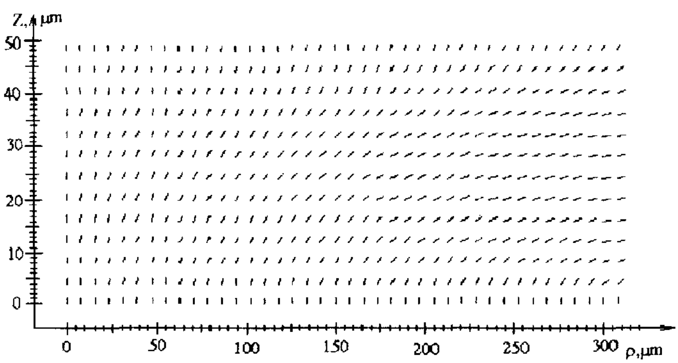


FIGURE 5 Nematic director distribution in LCM of diameter 600 μm at $U_p = 14\ V$.

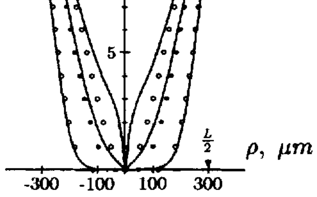
Distribution of nematic director obtained from equation (5) is illustrated in Figure 5. It's clear that distribution is inhomogeneous. Homeotropical alignment ($\theta = 0$) is observed on the microlens axis. Deformation is increasing at displacing to the edge. Distribution of the director becomes homogeneous out of the limits LCM.

COMPARING RESULTS OF CALCULATION AND EXPERIMENT

Profiles of phase delay and focal distances of microlens under different voltage are magnitudes measured experimentally. They were calculated numerically from distribution of nematic director and then were compared with experimentally measured data. The profiles of phase delay $\Delta\Phi(\rho)$ were calculated from the next equations:

$$\Delta\Phi(\rho) = \frac{2\pi}{\lambda}\left[\int_0^d n[\theta(\rho,z)]dz - n_o d\right], \quad n(\theta) = \frac{n_o n_e}{(n_e^2\cos^2\theta + n_o^2\sin^2\theta)^{1/2}},$$

where n_e - is the index of refraction of extraordinary wave.
Phase profiles for microlens of diameter $600\mu m$ under voltage 10, 14 and 16 V are shown on the Figure 6. The experimental values of phase delay profiles have been obtained earlier [1] and are shown by the points just here. As one can see from figure nematic deformation is increasing at increased voltages and deformation area is displacing near to the center of the microlens. Profiles of phase delay in microlens have close to quadratic dependence from radial coordinate in this voltage range. The microlens posseses defocusing properties. Deformation is beginning in the center of hole and just here defect forms at further increased voltages and phase profile looses quadratic character.



The calculated curves have certain accordance with experimental data in the center and near the edge of LCM at 15 V voltage and higher. The calculation gives less values than experimental data in the rest part of microlens. Calculations in this voltage range don't have practical importance in the given investigation of the electrical optical properties of LCM.

FIGURE 6 Profiles of phase delay for LCM of diameter 600 μm: experimental and calculation.

TABLE 1 The microlenses focal distances.

L/d	U_p, V	F_{calc}, mm	F_{exp}, mm
3,6	6	1,09	1,09
7,6	10	2,48	2,65
12	14	5,21	5,3
15,6	17	7,79	no data

The phase profile is described by parabola at voltage U_p:

$$\Delta\Phi(\rho) = \frac{2\pi}{\lambda}\frac{\rho^2}{2f}.$$

Focal distances f of LC-microlenses were calculated proceed from this equation. The results of focal distances calculation and comparing them with experimental data are represented in the next table. One can see, that satisfactory accordance of calculation results with experiments is observed.

CONCLUSION

The process of nematic realignment in inhomogeneous field of LCM at increased U passes five stages with different kinds of phase profile - with plateau in center ($U < U_p$); parabolic defocusing at $U = U_p$; conical; with disclination in center; spiralized. The calculation nematic deformation in axially symmetrical field of LCM was carried out at large deformation angles. It allowed to describe first two stages theoretically. The numerical agreement of calculation results with experiment was obtained. The focal distances under different voltages are calculated for various microlenses. A comparison of calculated values with experimental results has revealed satisfactory quantitive agreement for three microlenses at voltage U_p. The obtained results may be used at construction of optical devices based LCM (in fiber optics, as transformer of light polarization, in spatial systems).

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